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RULES FOR ROUNDING OFF NUMERICAL VALUES

(Revised)

(Incorporating Amendment Nos. 1, 2 & 3)

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BUREAU OF INDIAN STANDARDS
MANAK BHAVAN, 9 BAHADUR SHAH ZAFAR MARG
NEW DELHI 110002

Price Group 3

Indian Standard
RULES FOR ROUNDING OFF NUMERICAL VALUES
(Revised)

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Indian Standard

RULES FOR ROUNDING OFF NUMERICAL VALUES

(Revised)

0. FOREWORD

0.1 This Indian Standard (Revised) was adopted by the Indian Standards Institution on 27 July 1960, after the draft finalized by the Engineering Standards Sectional Committee had been approved by the Engineering Division Council.

0.2 To round off a value is to retain a certain number of figures, *counted from the left*, and drop the others so as to give a more rational form to the value. As the result of a test or of a calculation is generally rounded off for the purpose of reporting or for drafting specifications, it is necessary to prescribe rules for 'rounding off' numerical values as also for deciding on 'the number of figures' to be retained.

0.3 This standard was originally issued in 1949 with a view to promoting the adoption of a uniform procedure in *rounding off* numerical values. However, the rules given referred only to unit fineness of rounding (*see* **2.3**) and in course of years the need was felt to prescribe rules for rounding off numerical values to fineness of rounding other than unity. Moreover, it was also felt that the discussion on the number of figures to be retained as given in the earlier version required further elucidation. The present revision is expected to fulfil these needs.

0.4 In preparing this standard, reference has been made to the following:

IS : 787-1956 GUIDE FOR INTER-CONVERSION OF VALUES FROM ONE SYSTEM OF UNITS TO ANOTHER. Indian Standards Institution.

IS : 1890 (PART 0)-1995/ISO 31-0 : 1992 QUANTITIES AND UNITS : PART 0 GENERAL PRINCIPLES (*FIRST REVISION*).

BS 1957 : 1953 PRESENTATION OF NUMERICAL VALUES (FINENESS OF EXPRESSION; ROUNDING OF NUMBERS). British Standards Institution.

AMERICAN STANDARD Z 25.1-1940 RULES FOR ROUNDING OFF NUMERICAL VALUES. American Standards Association.

ASTM DESIGNATION : E 29-50 RECOMMENDED PRACTICE FOR DESIGNATING SIGNIFICANT PLACES IN SPECIFIED VALUES. American Society for Testing and Materials.

JAMES W. SCARBOROUGH. Numerical Mathematical Analysis Baltimore. The John Hopkins Press, 1955.

0.5 This edition 2.3 incorporates Amendment No. 1 (Feburary 1997), Amendment No. 2 (October 1997) and Amendment No. 3 (August 2000). Side bar indicates modification of the text as the result of incorporation of the amendments.

1. SCOPE

1.1 This standard prescribes rules for rounding off numerical values for the purpose of reporting results of a test, an analysis, a measurement or a calculation, and thus assisting in drafting specifications. It also makes recommendations as to the number of figures that should be retained in course of computation.

2. TERMINOLOGY

2.0 For the purpose of this standard, the following definitions shall apply.

2.1 Number of Decimal Places — A value is said to have as many decimal places as there are number of figures in the value, counting from the first figure after the decimal point and ending with the last figure on the right.

Examples:

Value	*Decimal Places*
0.029 50	5
21.029 5	4
2 000.000 001	6
291.00	2
10.32×10^3 (*see* Note 1)	2

NOTE 1 — For the purpose of this standard, the expression 10.32×10^3 should be taken to consist of two parts, the value proper which is 10.32 and the unit of expression for the value, 10^3.

2.2 Number of Significant Figures — A value is said to have as many significant figures as there are number of significant *digits* (*see* Note 2) in the value, counting from the left-most *non-zero* digit and ending with the right-most digit in the value.

Examples:

Value	*Significant Figures*
0.029 500	5
0.029 5	3
10.029 5	6
2 000.000 001	10
5 677.0	5
567 700	6
56.77×10^2	4
0 056.770	5
3 900 (*see* Note 3)	4

NOTE 2 — Any of the digits, 1, 2, 3,............9 occurring in a value shall be a significant digit(s); and zero shall be a significant digit only when it is preceded by some other digit (excepting zeros) on its left. When appearing in the power of 10 to indicate the magnitude of the unit in the expression of a value, zero shall not be a significant digit.

NOTE 3 — With a view to removing any ambiguity regarding the significance of the zeros at the end in a value like 3 900, it would be always desirable to write the value in the power-of-ten notation. For example, 3 900 may be written as 3.9×10^3, 3.90×10^3 or 3.900×10^3 depending upon the last figure(s) in the value to which it is desired to impart significance.

2.3 Fineness of Rounding — The unit to which a value is rounded off.

For example, a value may be rounded to the nearest 0.000 01, 0.000 2, 0.000 5, 0.001, 0.002 5, 0.005, 0.01, 0.07, 1, 2.5, 10, 20, 50, 100 or any other unit depending on the fineness desired.

3. RULES FOR ROUNDING

3.0 The rule usually followed in rounding off a value to unit fineness of rounding is to keep unchanged the last figure retained when the figure next beyond is less than 5 and to increase by 1 the last figure retained when the figure next beyond is more than 5. There is diversity of practice when the figure next beyond the last figure retained is 5. In such cases, some computers ‘round up’, that is, increase by 1, the last figure retained; others ‘round down’, that is, discard everything beyond the last figure retained. Obviously, if the retained value is always ‘rounded up’ or always ‘rounded down’, the sum and the average of a series of values so rounded will be larger or smaller than the corresponding sum or average of the unrounded values. However, if rounding off is carried out in accordance with the rules stated in **3.1** in one step (*see* **3.3**), the sum and the average of the rounded values would be more nearly correct than in the previous cases (*see* Appendix A).

3.1 Rounding Off to Unit Fineness — In case the fineness of rounding is unity in the last place retained, the following rules (except in **3.4**) shall be followed:

Rule I — When the figure next beyond the last figure or place to be retained is less than 5, the figure in the last place retained shall be left unchanged.

Rule II — When the figure next beyond the last figure or place to be retained is more than 5 or is 5 followed by any figures other than zeros, the figure in the last place retained shall be increased by 1.

Rule III — When the figure next beyond the last figure or place to be retained is 5 alone or 5 followed by zeros only, the figure in the last place retained shall be (a) increased by 1 if it is odd and (b) left unchanged if even (zero would be regarded as an even number for this purpose).

Some examples illustrating the application of Rules I to III are given in Table I.

TABLE I EXAMPLES OF ROUNDING OFF VALUES TO UNIT FINENESS

VALUE	FINENESS OF ROUNDING							
	1		0.1		0.01		0.001	
	Rounded Value	Rule	Rounded Value	Rule	Rounded Value	Rule	Rounded Value	Rule
7.260 4	7	I	7.3	II	7.26	I	7.260	I
14.725	15	II	14.7	I	14.72	III(b)	14.725	—
3.455	3	I	3.5	II	3.46	III(a)	3.455	—
13.545 001	14	II	13.5	I	13.55	II	13.545	I
8.725	9	II	8.7	I	8.72	III(b)	8.725	—
19.205	19	I	19.2	I	19.20	III(b)	19.205	—
0.549 9	1	II	0.5	I	0.55	II	0.550	II
0.650 1	1	II	0.7	II	0.65	I	0.650	I
0.049 50	0	I	0.0	I	0.05	II	0.050	III(a)

3.1.1 The rules for rounding laid down in **3.1** may be extended to apply when the fineness of rounding is 0.10, 10, 100, 1 000, etc. For example, 2.43 when rounded to fineness 0.10 becomes 2.40. Similarly, 712 and 715 when rounded to the fineness 10 become 710 and 720 respectively.

3.2 Rounding Off to Fineness Other than Unity — In case the fineness of rounding is not unity, but, say, it is n, the given value shall be rounded off according to the following rule:

Rule IV — When rounding to a fineness n, other than unity, the given value shall be divided by n. The quotient shall be rounded off to the nearest whole number in accordance with the rules laid down in **3.1** for unit fineness of rounding. The number so obtained, that is the rounded quotient, shall then be multiplied by n to get the final rounded value.

Some examples illustrating the application of Rule IV are given in Table II.

NOTE 4 — The rules for rounding off a value to any fineness of rounding, n, may also be stated in line with those for unit fineness of rounding (*see* **3.1**) as follows:

Divide the given value by n so that an integral quotient and a remainder are obtained. Round off the value in the following manner:

a) If the remainder is less than $n/2$, the value shall be rounded down such that the rounded value is an integral multiple of n.

b) If the remainder is greater than $n/2$, the value shall be rounded up such that the rounded value is an integral multiple of n.

c) If the remainder is exactly equal to $n/2$, that rounded value shall be chosen which is an integral multiple of $2n$.

TABLE II EXAMPLES OF ROUNDING OFF VALUES TO FINENESS OTHER THAN UNIT

VALUE	FINENESS OF ROUNDING, n	QUOTIENT	ROUNDED QUOTIENT	FINAL ROUNDED VALUE
(1)	(2)	(3) = (1)/(2)	(4)	(5) = (2) × (4)
1.647 8	0.2	8.239	8	1.6
2.70	0.2	13.5	14	2.8
2.496 8	0.3	8.322 7	8	2.4
1.75	0.5	3.5	4	2.0
0.687 21	0.07	9.817 3	10	0.70
0.875	0.07	12.5	12	0.84
325	50	6.5	6	3×10^2
1 025	50	20.5	20	10×10^2

3.2.1 Fineness of rounding other than 2 and 5 is seldom called for in practice. For these cases, the rules for rounding may be stated in simpler form as follows:

a) Rounding off to fineness 50, 5, 0.5, 0.05, 0.005, etc.

Rule V — When rounding to 5 units, the given value shall be doubled and rounded off to twice the required fineness of rounding in accordance with **3.1.1**. The value thus obtained shall be halved to get the final rounded value.

For example, in rounding off 975 to the nearest 50, 975 is doubled giving 1 950 which becomes 2 000 when rounded off to the nearest 100; when 2 000 is divided by 2, the resulting number 1 000 is the rounded value of 975.

b) Rounding off to fineness 20, 2, 0.2, 0.02, 0.002, etc.

Rule VI — When rounding to 2 units, the given value shall be halved and rounded off to half the required fineness of rounding in accordance with **3.1**. The value thus obtained shall then be doubled to get the final rounded value.

For example, in rounding off 2.70 to the nearest 0.2, 2.70 is halved giving 1.35 which becomes 1.4 when rounded off to the nearest 0.1; when 1.4 is doubled, the resulting number 2.8 is the rounded value.

3.3 Successive Rounding — The final rounded value shall be obtained from the most precise value available in one step only and not from a series of successive roundings. For example, the value 0.549 9, when rounded to one significant figure, shall be written as 0.5 and not as 0.6 which is obtained as a result of successive roundings to 0.550, 0.55, and 0.6. It is obvious that the most precise value available is nearer to 0.5 and not to 0.6 and that the error involved is less in the former case. Similarly, 0.650 1 shall be rounded off to 0.7 in one step and not successively to 0.650, 0.65 and 0.6, since the most precise value available here is nearer to 0.7 than to 0.6 (*see also* Table I).

NOTE 5 — In those cases where a final rounded value terminates with 5 and it is intended to use it in further computation, it may be helpful to use a '+' or '–' sign after the final 5 to indicate whether a subsequent rounding should be up or down. Thus 3.214 7 may be written as 3.215– when rounded to a fineness of rounding 0.001. If further rounding to three significant figures is desired, this number would be rounded down and written as 3.21 which is in error by *less* than half a unit in the last place; otherwise, rounding of 3.215 would have yielded 3.22 which is in error by *more* than half a unit in the last place. Similarly, 3.205 4 could be written as 3.205+ when rounded to 4 significant figures. Further rounding to 3 significant figures would yield the value as 3.21.

In case the final 5 is obtained exactly, it would be indicated by leaving the 5 as such without using '+' or '–' sign. In subsequent rounding the 5 would then be treated in accordance with Rule III.

3.4 The rules given in **3.1**, **3.2** and **3.3** should be used only if no specific criteria for the selection of the rounded number have to be taken into account. In all cases, where safety requirements or prescribed limits have to be respected, rounding off should be done in one direction only.

4. NUMBER OF FIGURES TO BE RETAINED

4.0 Pertinent to the application of the rules for rounding off is the underlying decision as to the number of figures that should be retained in a given problem. The original values requiring to be rounded off may arise as a result of a test, an analysis or a measurement, in other words, experimental results, or they may arise from computations involving several steps.

4.1 Experimental Results — The number of figures to be retained in an experimental result, either for the purpose of reporting or for guiding the formulation of specifications will depend on the significance of the figures in the value. This aspect has been discussed in detail under **4** of IS : 787-1956 to which reference may be made for obtaining helpful guidance.

4.2 Computations — In computations involving values of different accuracies, the problem as to how many figures should be retained at various steps assumes a special significance as it would affect the accuracy of the final result. The rounding off error will, in fact, be injected into computation every time an arithmetical operation is performed. It is, therefore, necessary to carry out the computation in such a manner as would obtain accurate results consistent with the accuracy of the data in hand.

4.2.1 While it is not possible to prescribe details which may be followed in computations of various types, certain basic rules may be recommended for single arithmetical operations which, when followed, will save labour and at the same time enable accuracy of original data to be normally maintained in the final answers.

4.2.2 As a guide to the number of places or figures to be retained in the calculations involving arithmetical operations with rounded or approximate values, the following procedures are recommended:

a) *Addition* — The more accurate values shall be rounded off so as to retain *one more place* than the last significant figure in the least accurate value. The resulting sum shall then be rounded off to the last significant place in the least accurate value.

b) *Subtraction* — The more accurate value (of the two given values) shall be rounded off, before subtraction, to the *same place* as the last significant figure in less accurate value; and the result shall be reported as such (*see also* Note 6).

c) *Multiplication and Division* — The number of *significant figures* retained in the more accurate values shall be kept *one more* than that in the least accurate value. The result shall then be rounded off to the same number of significant figures as in the least accurate value.

d) When a long computation is carried out in several steps, the intermediate results shall be properly rounded at the end of each step so as to avoid the accumulation of rounding errors in such cases. It is recommended that, at the end of each step, one more significant figure may be retained than is required under (a), (b) and (c) (*see also* Note 7).

NOTE 6 — The loss of the significant figures in the subtraction of two nearly equal values is the greatest source of inaccuracy in most computations, and it forms the weakest link in a chain computation where it occurs. Thus, if the values 0.169 52 and 0.168 71 are each correct to five significant figures, their difference 0.000 81, which has only two significant figures, is quite likely to introduce inaccuracy in subsequent computation.

If, however, the difference of two values is desired to be correct to k significant figures and if it is known beforehand that the first m significant figures at the left will disappear by subtraction, then the number of significant figures to be retained in each of the values shall be $m + k$ (*see* Example 4).

NOTE 7 — To ensure a greater degree of accuracy in the computations, it is also desirable to avoid or defer as long as possible certain approximation operations like that of the division or square root. For example, in the determination of sucrose by volumetric method, the expression $\frac{20 w_1}{w_2}\left(\frac{f_2}{v_2} - \frac{f_1}{v_1}\right)$ may be better evaluated by taking its calculational form as $20w_1 (f_2 v_1 - f_1 v_2)/w_2 v_1 v_2$ which would defer the division until the last operation of the calculation.

4.2.3 *Examples*

Example 1

Required to find the sum of the rounded off values 461.32, 381.6, 76.854 and 4.746 2.

Since the least accurate value 381.6 is known only to the first decimal place, all other values shall be rounded off to one more place, that is, to two decimal places and then added as shown below:

$$
\begin{array}{r}
461.32 \\
381.6 \\
76.85 \\
4.75 \\
\hline
924.52
\end{array}
$$

The resulting sum shall then be reported to the same decimal place as in the least accurate value, that is, as 924.5.

Example 2

Required to find the sum of the values 28 490, 894, 657.32, 39 500 and 76 939, assuming that the value 39 500 is known to the nearest hundred only.

Since one of the values is known only to the nearest hundred, the other values shall be rounded off to the nearest ten and then added as shown below:

$$
\begin{array}{r}
2\ 849 \times 10 \\
89 \times 10 \\
66 \times 10 \\
3\ 950 \times 10 \\
7\ 694 \times 10 \\
\hline
14\ 648 \times 10
\end{array}
$$

The sum shall then be reported to the nearest hundred as 1 465 × 100 or even as 1.465×10^5.

Example 3

Required to find the difference of 679.8 and 76.365, assuming that each number is known to its last figure but no farther.

Since one of the values is known to the first decimal place only, the other value shall also be rounded off to the first decimal place and then the difference shall be found.

$$\begin{array}{r} 679.8 \\ 76.4 \\ \hline 603.4 \end{array}$$

The difference, 603.4, shall be reported as such.

Example 4

Required to evaluate $\sqrt{2.52} - \sqrt{2.49}$ correct to five significant figures.

Since $\sqrt{2.52} = 1.587\ 450\ 79$

$\sqrt{2.49} = 1.577\ 973\ 38$

and three significant figures at the left will disappear on subtraction, the number of significant figures retained in each value shall be 8 as shown below:

$$\begin{array}{r} 1.587\ 450\ 8 \\ 1.577\ 973\ 4 \\ \hline 0.009\ 477\ 4 \end{array}$$

The result, 0.009 477 4, shall be reported as such (or as $9.477\ 4 \times 10^{-3}$).

Example 5

Required to evaluate $35.2/\sqrt{2}$, given that the numerator is correct to its last figure.

Since the numerator here is correct to three significant figures, the denominator shall be taken as $\sqrt{2} = 1.414$. Then,

$$\frac{35.2}{1.414} = 24.89$$

and the result shall be reported as 24.9.

Example 6

Required to evaluate $3.78\pi/5.6$, assuming that the denominator is true to only two significant figures.

Since the denominator here is correct to two significant figures, each number in the numerator would be taken up to three significant figures. Thus,

$$\frac{3.78 \times 3.14}{5.7} = 2{,}08.$$

The result shall, however, be reported as 2.1.

APPENDIX A

(Clause 3.0 *)*

VALIDITY OF RULES

A-1. The validity of the rules for rounding off numerical values, as given in **3.1**, may be seen from the fact that to every number that is to be 'rounded down' in accordance with Rule I, there corresponds a number that is to be 'rounded up' in accordance with Rule II. Thus, these two rules establish a balance between rounding 'down' and 'up' for all numbers other than those that fall exactly midway between two alternatives. In the latter case, since the figure to be dropped is exactly 5, Rule III, which specifies that the value should be rounded to its nearest even number, implies that rounding shall be 'up' when the preceding figures are 1, 3, 5, 7, 9 and 'down' when they are 0, 2, 4, 6, 8. Rule III hence advocates a similar balance between rounding 'up' and 'down' (*see also* Note 8). This implies that if the above rules are followed in a large group of values in which random distribution of figures occurs, the number 'rounded up' and the number 'rounded down' will be nearly equal. Therefore, the sum and the average of the rounded values will be more nearly correct than would be the case if all were rounded in the same direction, that is, either all 'up' or all 'down'.

NOTE 8 — From purely logical considerations, a given value could have as well been rounded to an odd number (and not an even number as in Rule III) when the discarded figures fall exactly midway between two alternatives. But there is a practical aspect to the matter. The rounding off a value to an even number facilitates the division of the rounded value by 2 and the result of such division gives the correct rounding of half the original unrounded value. Besides, the (rounded) even values may generally be exactly divisible by many more numbers, even as well as odd, than are the (rounded) odd values.

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Amendments are issued to standards as the need arises on the basis of comments. Standards are also reviewed periodically; a standard along with amendments is reaffirmed when such review indicates that no changes are needed; if the review indicates that changes are needed, it is taken up for revision. Users of Indian Standards should ascertain that they are in possession of the latest amendments or edition by referring to the latest issue of 'BIS Catalogue' and 'Standards : Monthly Additions'.

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